A perpetual principle for the medium of exchange accountability

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A perpetual principle for the unit of account would turn the medium of exchange as we currently have into a timeless concept. The global weighted sum of all assets would become the tangency point as the standard unit of money. All internationally applied mediums of exchange have their value derived from subjective concepts. Even with the backing of assets guaranteeing their value, they present a low degree of *conceptual infinity*. Part of the solution is provided by transitioning the existing units of account into the mean-variance efficient portfolio. Theoretically, it might seem easier to achieve a *tangency portfolio*. However, in this case, we are dealing with a *tangency medium of exchange* as the standard unit of account as a means of exchange for goods and services. Transactions will include the parts considered hard to account for. Additionally, as the concept remains intact the tangency medium of exchange is an ideal store of wealth which can be transmitted to any other time in history.

1. Introduction

When analyzing the history of money, it is evident that money is a product of the free market, and the most sought currency is a design of the ‘’invisible hand’’. By the time Nicolaus Copernicus turned his attention to monetary affairs, he developed the so-called *quantity theory of money*. His findings regarding the circulating gold coins were not different from other commodities, an increase in supply eventually causes the current value of each unit to go down as individuals have their *share diluted*. In his findings, there was an anomalous phenomenon causing the general prices to go up, *debasement*. Debasement was done by simply adding a minor portion of less valuable metal inside the gold coins and melting them together. The main purpose of having a medium of exchange is to facilitate transactions and lower their costs. Money has evolved spontaneously over the course of economic history. Money is a common measure of values. It must always preserve a fixed standard. The issue with all units of account is the subjectivity added to it. The replication of thinking in the future indicates the present flaw on the status. Past subjectivity does not correlate with identical future value attributions.

An internationally accepted medium of exchange, as we currently have, is a natural product of the market. It has evolved to its current state due to the quality of the commodity held by investors, its exchangeability to a broader range of commodities and its superiority as a store of

value. The store of value perception is attached to the conceptual infinity of an asset, how long can subjectivity play a role of value? Conceptual infinity meaning how long the value of something can last subjectively and objectively without losing its significance. The spontaneous design of a currency intrinsically ponders the value it holds, its conceptual infinity as a store of value and the characteristics implied on a commodity with high exchange value. The valuableness or, value in exchange, of a commodity intrigued many economists to study the causes to why certain commodities might have high *exchange value* but approximately zero *value in use*. Marking the history of economic thought, we can assume the *diamond-water paradox* as the most confrontational matter which formed vital conclusions. In this paper, we will focus on the objective characteristics an internationally applied medium of exchange ought to have, the degrees of conceptual infinity, high values in exchange and what contributions can be made to turn a high exchange value commodity to a high in use value one.

The degree of easiness in which a commodity can be exchanged for another was one of the first attributes sought by merchants when replacing not so liquid commodities. The tendency continues to investment concepts nowadays. We prefer more liquid investments which can be converted to the commodity with the highest degree of acceptance, cash in this case. The exchange value of goods is the equivalent to other goods, better stating, their equivalent in specified conditions to secure a precise amount of other goods as a comparison in exchange. As an initial point for merchants or investors the preferred ground is the one with more individuals on it. The ideal currency has characteristics that will not become obsolete subjectively speaking. There was a time where salt was a medium of exchange, nowadays we purchase salt with the most accepted medium of exchange in the market. As we experience new conducts and technology development, efficient economic ways of interacting are more concerned with durability. Money's applicability as a store of value ought to give confidence to the investors and frequent users, *a credible intellectual commitment*. The bases of money as a medium of exchange most likely become a *commodity* base system, any commodity might eventually become obsolete if it only holds a subjective value perspective to it and little value in use.

Although there are several ways money has evolved, the value exchange of money has become hard to calculate and how it stabilizes as a storage of wealth. The principles circulating the standard of money's value have become peculiar. These principles are debilitating the

exchange value of money. The exchange of goods valued at different exchangeable currencies imply both different objective and subjective values once a decision is made based on the conditions of time and physical location. The idea of measuring variations in prices and quantities demanded based on different locations, more precisely, purchasing power parity (PPP) originated in the 16th century. The rate variations can be equalized for calculation purposes and the purchasing power of money. The School of Salamanca was the first one to address the issue. Intrigued by the discrepancies in price at different locations, the causes of this issue were examined by PPP concept. The PPP is constructed on the law of one price, in the absence of transaction costs and trade barriers, equal goods will have equal price in diverse markets once the prices are stated in the equivalent currency.

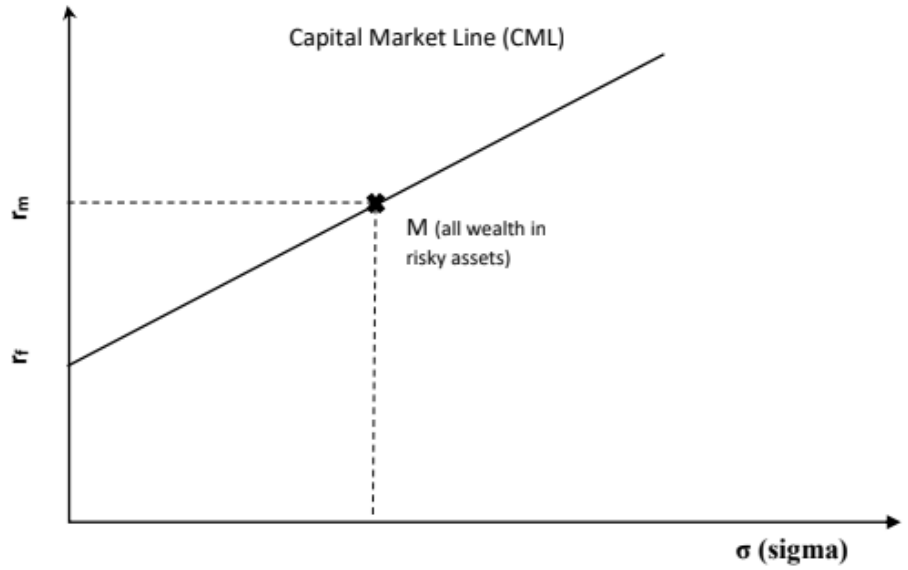
2. Impartial Unit of Account

The base value of money, as a principle, ought to have a stationary and impartial characteristic. This characteristic of money must consider its universality and application. The future is always unpredictable, making it impossible to forward base economic calculation with certainty. Future expected returns have an easier calculation assessment as it acknowledged in the capital markets. The global weighted sum of all assets being the substance of every unit of account present in the market is the universal piece required for economic development and calculation. *Cash* as a global weighted sum of every asset is a way to lock the purchasing power and hedge the risks of inflation. Its characteristics are perpetual, and it becomes the foundation to any future projection.

One common critique when applying the weighted sum of all world's assets would be the lack of precision when it comes to accounting the non-liquid assets (or assets that are not publicly traded such as private property, jewelry, human capital, etc.). This critique is somehow valid when considering it as portfolio like investment without the network effect, however, not valid for the role medium of exchange. The Dollarization of assets is the likely phenomenon to take place with the accounting of cash as global market weighted assets (GMP).

The market portfolio is the portfolio consisting of a weighted sum of every asset in the market, with weights in the proportions that they exist in the market. The CML is the tangent line drawn from the risk-free point to the feasible region for risky assets. The tangency point M between Capital Market Line (CML) and efficient frontier represents Market Portfolio.

Equation of CML: $r_p = r_f + \frac{r_m - r_f}{\sigma_m}\,\sigma_P$



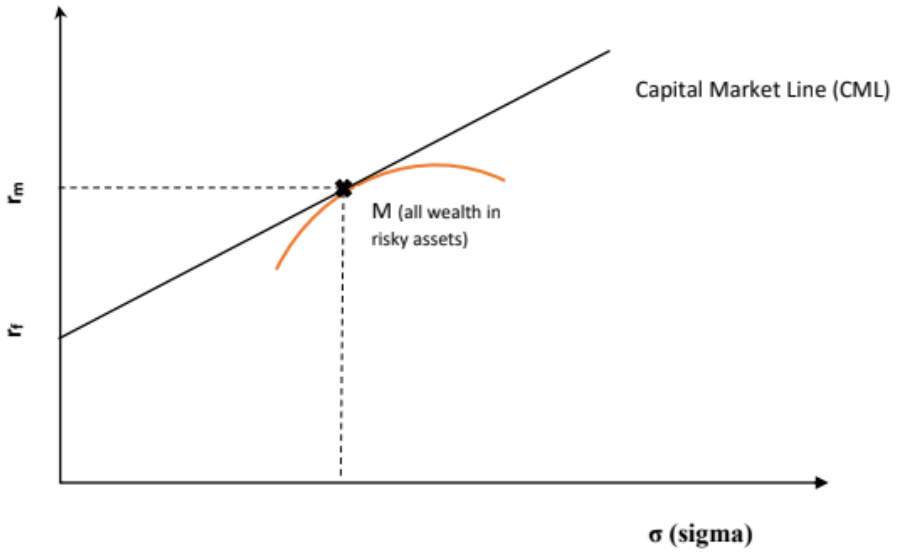
Consider the portfolio with α percent invested in asset *i* and 1- α percent in the market portfolio M. The expected return and risk of the portfolio can be expressed mathematically as,

$$r_\alpha = \alpha\, r_i + (1-\alpha)\, r_m \qquad (1)$$

And its variance,

$$\sigma_\alpha^2 = \alpha^2\, \sigma_i^2 + 2\alpha(1-\alpha)\, \sigma_{im} + (1-\alpha)^2 \sigma_m^2 \qquad (2)$$

By taking different values of α, efficient frontier traced out by (σ_α , r_α)



Taking partial derivative on both sides of equation (1) and (2)

$$\frac{\partial r_\alpha}{\partial \alpha} = r_i - r_m$$

$$\frac{\partial \sigma_\alpha}{\partial \alpha} = \frac{\alpha\sigma_i^2 + (1-2\alpha)\sigma_{im} + (\alpha-1)\sigma_m^2}{\sigma_\alpha}$$

For market portfolio $\alpha = 0$, above equation become,

$$\frac{\partial \sigma_\alpha}{\partial \alpha} = \frac{\sigma_{im} - \sigma_m^2}{\sigma_m}$$

Using chain rule,

$$\frac{\partial r_\alpha}{\partial \sigma_\alpha} = \frac{\frac{\partial r_\alpha}{\partial \alpha}}{\frac{\partial \sigma_\alpha}{\partial \alpha}}$$

$$\frac{\partial r_\alpha}{\partial \sigma_\alpha} = \frac{r_i - r_m}{\frac{\sigma_{im} - \sigma_m^2}{\sigma_m}}$$

$$\frac{\partial r_\alpha}{\partial \sigma_\alpha} = \sigma_m \frac{r_i - r_m}{\sigma_{im} - \sigma_m^2}$$

Since the tangency point M between the efficient frontier and CML represents the market portfolio, therefore, equating slopes of efficient frontier and CML,

$$\frac{r_m - r_f}{\sigma_m} = \sigma_m \frac{r_i - r_m}{\sigma_{im} - \sigma_m^2}$$

$$\frac{\sigma_{im} - \sigma_m^2}{\sigma_m^2} = \frac{r_i - r_m}{r_m - r_f}$$

$$\frac{\sigma_{im}}{\sigma_m^2} - \frac{\sigma_m^2}{\sigma_m^2} = \frac{r_i - r_m}{r_m - r_f}$$

$$\beta - 1 = \frac{r_i - r_m}{r_m - r_f}$$

$$\boxed{r_m = \frac{r_i + r_f(\beta - 1)}{\beta}}$$

This formula represents the possibility of all risky assets in the world combined by their shared proportion.

3. **Cash-GMP** (Global weighted sum of all assets) **Denomination**

Suppose there are *n* different risky assets available in the market with a column vector ***r.*** A portfolio return is $r_p = \boldsymbol{\alpha}^T \boldsymbol{r}$, where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$ is the proportion of the portfolio invested in asset *i*. A traditional mean-variance (i.e. with no cash) solves the following optimization problem[1]:

$$\min \boldsymbol{\alpha}^T \boldsymbol{V} \boldsymbol{\alpha} \tag{1}$$

subject to;

constraint#1: $\boldsymbol{\alpha}^T \boldsymbol{E(r)} = \boldsymbol{\mu}$,

constraint#2: $\boldsymbol{\alpha}^T \mathbf{1} = 1$,

Where $\boldsymbol{V} = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_n^2 \end{bmatrix}$ is the variance-covariance matrix of asset returns. μ represents the expected return on the portfolio, while **1** is the *n*-column vector with all entities equal to 1. The second constraint implies that all funds are invested, and no cash is retained. Using Lagrange-multiplier we can obtain the following solution of the above problem:

$$\alpha_{MV} = \frac{r_1 - \mu}{r_1 - r_0} \alpha_0 + \frac{\mu - r_0}{r_1 - r_0} \alpha_1 \tag{2}$$

Where α_0 represents weight corresponding to the minimum-variance portfolio while α_1 represents weight corresponding to point of intersection the efficient frontier and straight line passing through the origin.

The traditional mean-variance approach (above) does not reflect the fact that investors are able to keep a portion of their fund in cash account

A medium of exchange with such accountability would be the vital characteristic of any transaction making the non-liquid assets accounted at the end sum of the product in order to bring the non-liquid assets into the tangency point of the global portfolio.

4. **Universal Unit of Account and Risk Parity**

The risk-parity approach defines a well-diversified portfolio in which all assets have the same marginal contribution of the total risk of the portfolio. Therefore, such a portfolio is an equally weighted portfolio where the weights refer to risk rather than the actual amount invested in each

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[1] $\boldsymbol{\alpha}^T \boldsymbol{V} \boldsymbol{\alpha}$ is the vector representation of portfolio variance
$\boldsymbol{\alpha}^T \boldsymbol{E(r)}$ is the vector representation of portfolio return
$\boldsymbol{\alpha}^T \mathbf{1}$=1 is the vector representation of $\sum_{i=1}^{n} \alpha_i = 1$
Note that any representation in bold is representing vector quantity

asset. The general definition of the marginal contribution of an asset *i* to the total risk of the portfolio can be defined as:

$$MC_i = (\text{Weight of Asset } i) \times \frac{\Delta\, Total\ risk\ of\ Portfolio}{\Delta\, Weight\ of\ Asset\ Class\ i}$$

Alternatively,

$$MC_i = w_i \times \frac{\partial \sigma_p}{\partial w_i}$$

Here MC_i is the marginal contribution of risk of asset class *i* to the total risk. $\frac{\partial \sigma_p}{\partial w_i}$ is the change in a portfolio risk due to a very small change in weight of asset *i*. In other words, marginal contribution shows how much each asset contributes to the total risk and how this contribution varies when the weight of asset i changes. Total risk of the portfolio is the sum of the marginal contribution of the risk of each asset.

$$Total\ Risk = MC_1 + MC_2 + \cdots + MC_n$$

To see how it works, let's consider two assets only. The expected return and risk of the portfolio can be written as:

$$r_p = w_1 r_1 + w_2 r_2$$

$$\sigma_p = \sqrt{w_1^2\, \sigma_1^2 + w_2^2\, \sigma_2^2 + 2 w_1 w_2 Cov\,(r_1, r_2)}$$

Where w_1 & w_2, σ_1 & σ_2, cov(r_1, r_2) are the weights, standard deviations, and covariance respectively. Weight of an asset is inversely related to the standard deviation i.e. an asset would be of least weightage if it has highest risk. It maintains equal marginal contribution of risk of each asset to the portfolio risk.

$$w_1 = \frac{\frac{1}{\sigma_1}}{\frac{1}{\sigma_1} + \frac{1}{\sigma_2}}$$

$$w_2 = \frac{\frac{1}{\sigma_2}}{\frac{1}{\sigma_1} + \frac{1}{\sigma_2}}$$

Weights are adjusted according to relative risk then change in portfolio risk due to change in weight can be expressed mathematically as,

$$\frac{\partial \sigma_p}{\partial w_1} = \frac{w_1\, \sigma_1^2 + w_2 Cov\,(r_1, r_2)}{\sigma_p}$$

$$\frac{\partial \sigma_p}{\partial w_2} = \frac{w_2\, \sigma_2^2 + w_1 Cov\,(r_1, r_2)}{\sigma_p}$$

5. Conclusion

What would happen if our medium of exchange had a perpetual principle? The Cash-GMP assessment addresses well the separation of a unit of account with perpetual value (the ideal representation of all risky assets according to their market weight) and the viability of a commodity as a medium of exchange only. The initial calculation for any future projection would have basis in something concrete and objective. Capital goods for example could evolve perpetually if they were hedged on the capital markets with a timeless and easily accessed unit of account. Other mediums of exchange present several attributes that validate them as an *accepted* medium of exchange, mostly the confidence in a specific institution or a subjective point of value attached to a commodity. The store of value role is one of the main characteristics that validated the idea of money. In modern times, the accountability of risky factors is examined with enormous precision once every data detail can be easily access with computer power. Considering the risks, volatility has a major significance when agents are allocating their capital in some sort of medium of exchange. The Cash-GMP assessment is further improved considering the balance in every asset class according to their volatility, risk parity.

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